## **Theory of Vibration/Shock Isolators**

The solutions to most isolator problems begin with consideration of the mounted system as a damped, single degree of freedom system. This allows simple calculations of most of the parameters necessary to decide if a standard isolator will perform satisfactorily or if a custom design is required. This approach is based on the facts that:

1. Many isolation systems involve center-of-gravity installations of the equipment. That is, the center-ofgravity of the equipment coincides with the elastic center of the isolation system. The center-of-gravity installation is often recommended since it allows performance to be predicted more accurately and it allows the isolators to be loaded in an optimum manner. Figure 1 shows some typical center-of-gravity systems.

2. Many equipment isolation systems are required to be isoelastic. That is, the system translational spring rates in all directions are the same.

3. Many pieces of equipment are relatively light in weight and support structures are relatively rigid in comparison to the stiffness of the isolators used to support and protect the equipment.



FIGURE 1 TYPICAL CENTER-OF-GRAVITY INSTALLATIONS

For cases which do not fit the above conditions, or where more precise analysis is required, there are computer programs available to assist the analyst. Lord computer programs for dynamic analysis are used to determine the system response to various dynamic disturbances. The loads, motions, and accelerations at various points on the isolated equipment may be found and support structure stiffnesses may be taken into account. Some of the more sophisticated programs may even accept and analyze nonlinear systems. This discussion is reason to emphasize the need for the information regarding the intended application of the isolated equipment. The dynamic environment, the ambient environment and the physical characteristics of the system are all important to a proper analysis. The use of the checklist included with this catalog is recommended as an aid.

With the above background in mind, the aim of this theory section will be to use the single degree-of freedom basis for the initial selection of standard isolators. This is the first step toward the design of custom isolators and the more complex analyses of critical applications.

#### SINGLE DEGREE-OF-FREEDOM DYNAMIC SYSTEM

Figure 2 shows the "classical" spring, mass, damper depiction of a single degree-of-freedom dynamic system. Figure 3 and the related equations show this system as either damped or undamped. Figure 4 shows the resulting vibration response transmissibility curves for the damped and undamped systems of Figure 3.

These figures and equations are well known and serve as a useful basis for beginning the analysis of an isolation problem. However, classical vibration theory is based on one assumption that requires understanding in the application of the theory. That assumption is that the properties of the elements of the system behave in a linear, constant manner. Data to be presented later will give an indication of the factors which must be considered when applying the analysis to the real world.



M—Mass—Stores kinetic energy K—Spring—Stores potential energy, supports load C—Damper—Dissipates energy, cannot support load

FIGURE 2 ELEMENTS OF A VIBRATORY SYSTEM





Damped



FIGURE 3 DAMPED AND UNDAMPED SINGLE DEGREE-OF-FREEDOM BASE EXCITED VIBRATORY SYSTEMS

The equations of motion for the above model systems are familiar to many. For review purposes, they are presented here.

#### FOR THE UNDAMPED SYSTEM

The differential equation of motion is:

$$\mathbf{M}\mathbf{X} + \mathbf{K}\mathbf{X} = \mathbf{F}(\mathbf{t})$$

In which it may be seen that the forces due to the dynamic input (which varies as a function of time) are balanced by the inertial force of the accelerating mass and the spring force. From the solution of this equation, comes the equation defining the natural frequency of an undamped spring-mass system:

$$\mathbf{f}_{n} = \frac{1}{2\pi} \sqrt{\mathbf{K}/\mathbf{M}}$$

Another equation which is derived from the solution of the basic equation of motion for the undamped vibratory system is that for transmissibility—the amount of vibration transmitted to the isolated equipment through the mounting system depending on the characteristics of the system and the vibration environment.

$$T_{ABS} = \frac{1}{(1-r^2)}$$

Wherein, "r" is the ratio of the exciting vibration frequency to the system natural frquency. That is:

$$r = \frac{f}{f_n}$$

In a similar fashion, the damped system may be analyzed. The equation of motion here must take into account the damper which is added to the system. It is:

$$\mathbf{M} \mathbf{X} + \mathbf{C} \mathbf{X} + \mathbf{K} \mathbf{X} = \mathbf{F}(\mathbf{t})$$

The equation for the natural frequency of this system may, for normal amounts of damping, be considered the same as for the undamped system. That is,

$$f_n = \frac{1}{2\pi} \sqrt{K/M}$$

In reality, the natural frequency does vary slightly with the amount of damping in the system. The damping factor is given the symbol " $\zeta$ " and is approximately one-half the loss factor, " $\eta$ ," described in the definition section regarding damping in elastomers. The equation for the natural frequency of a damped system, as related to that for an undamped system, is:

$$\mathbf{f}_{nd} = \mathbf{f}_n \sqrt{1 - \zeta^2}$$

The damping ratio,  $\zeta$ , is defined as:

$$\zeta = \mathbf{C} / \mathbf{C}_{\mathbf{c}}$$
$$\zeta \approx \eta / 2$$

**RPM Mechanical Inc** 

Website: www.rpmmech.com

Phone: (888) 842-5668

Where, the "critical" damping level for a damped vibratory system is defined as:

$$C_c = 2\sqrt{KM}$$

The equation for the absolute transmissibility of a damped system is written as:

$$T_{ABS} = \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{(2\zeta r)^2 + [(1 - r^2)]^2}}$$

The equations for the transmissibilities of the undamped and damped systems are plotted in Figure 4. As may be seen, the addition of damping reduces the amount of transmitted vibration in the amplification zone, around the natural frequency of the system ( $\mathbf{r} = \mathbf{1}$ ). It must also be noted that the addition of damping reduces the amount of protection in the isolation region (where  $\mathbf{r} > \sqrt{2}$ ).



FIGURE 4 TYPICAL TRANSMISSIBILITY CURVES

In the real world of practical isolation systems, the elements are not linear and the actual system response does not follow the above analysis rigorously. Typically, elastomeric isolators are chosen for most isolation schemes. Elastomers are sensitive to the vibration level, frequency and temperature to which they are exposed. The following discussion will present information regarding these sensitivities and provide some guidance in the application of isolators for typical installations.

#### **Elastomers for Vibration and Shock Isolation**

Depending on the ambient conditions and loads, a number of elastomers may be chosen for the isolators in a given isolation system. As seen in the above discussion, the addition of damping allows more control of the system in the region of resonance. The compromise which is made here though is that isolation is sacrificed. The higher the amount of damping, the greater the compromise. In addition, typical highly damped elastomers exhibit poor returnability and greater drift than elastomers which have medium or low damping levels. The requirements of a given application must be carefully weighed in order to select the appropriate elastomer.

Within the various families of Lord products, a number of elastomers may be selected. Some brief descriptions may help to guide in their selection for a particular problem.

**Natural Rubber** — This elastomer is the baseline for comparison of most others. It was the first elastomer and has some desirable properties, but also has some limitations in many applications. Natural rubber has high strength, when compared to most synthetic elastomers. It has excellent fatigue properties and low to medium damping which translates into efficient vibration isolation. Typically, natural rubber is not very sensitive to vibration amplitude (strain). On the limitation side, natural rubber is restricted to a fairly narrow temperature range for its applications. Although it remains flexible at relatively low temperatures, it does stiffen significantly at temperatures below 0°F. At the high temperature end, natural rubber is often restricted to use below approximately 180°F.

**Neoprene** — This elastomer was originally developed as a synthetic replacement for natural rubber and has nearly the same application range. Neoprene has more sensitivity to strain and temperature than comparable natural rubber compounds.

**SPE**<sup>®</sup>**I** — This is another synthetic elastomer which has been specially compounded by Lord for use in applications requiring strength near that of natural rubber, good low temperature flexibility and medium damping. The major use of SPE I elastomer has been in vibration and shock mounts for the shipping container industry. This material has good retention of flexibility to temperatures as low as -65°F. The high temperature limit for SPE I elastomer is typically +165°F.

**BTR<sup>®</sup>** — This elastomer is Lord's original "Broad Temperature Range" elastomer. It is a silicone elastomer which was developed to have high damping and a wide span of operational temperatures. This material has an application range from -65°F to +300°F. The loss factor of this material is in the range of 0.32. This elastomer has been widely used in isolators for Military Electronics equipment for many years. It does not have the high load carrying capability of natural rubber but is in the high range for materials with this broad temperature range.

**BTR II<sup>®</sup>**— This material is similar in use to the  $BTR^{\textcircled{R}}$  elastomer except that it has a slightly more limited temperature range and has less damping. BTR II may be used for most applications over a temperature range from  $-40^{\circ}$ F to  $+ 300^{\circ}$ F. The loss factor for typical BTR II compounds is in the range of 0.18. This elastomer has better returnability, less drift, and better stability with temperature, down to -40°F. The compromise with BTR II elastomer is the lower damping. This means that the resonant transmissibility of a system using BTR II elastomeric isolators will be higher than one using BTR isolators. At the same time, the high frequency isolation will be slightly better with the BTR II. This material has found use in Military Electronics isolators as well as in isolation systems for aircraft engines and shipboard equipment.

**BTR VI** — This is a very highly damped elastomer. It is a silicone elastomer of the same family as the BTR elastomer described above but is specially compounded to have loss factors in the 0.60 to 0.70 range. This would result in resonant transmissibility readings below 2.0 if used in a typical isolation system. This material is not used very often in applications requiring vibration isolation. It is most often used in products which are specifically designed for damping, such as lead-lag dampers for helicopter rotors. If used for a vibration isolator, BTR VI will provide excellent control of resonance but will not provide the degree of high frequency isolation that other elastomers will provide. The compromises here are that this material is quite strain and temperature sensitive, when compared to BTR and other typical Miltronics elastomers, and that it tends to have higher drift than the other materials.

**"MEM"** — This is an elastomer which has slightly less damping than Lord's BTR<sup>®</sup> silicone, but which also has less temperature and strain sensitivity. The typical loss factor for the MEM series of silicones is 0.29, which translates into a typical resonant transmissibility of 3.6 at room temperature and moderate strain across the elastomer. This material was developed by Lord at a time when some electronic guidance systems began to require improved performance stability of isolation systems across a broad temperature range, down to -70°F, while maintaining a reasonable level of damping to control resonant response.

**"MEA"** — With miniaturization of electronic instrumentation, equipment became slightly more rugged and could withstand somewhat higher levels of vibration, but still required more constant isolator performance over a wide temperature range. These industry trends led to the development of Lord MEA silicone. As may be seen in the material property graphs of Figures 5 through 8, this elastomer family offers significant improvement in strain and temperature sensitivity over the BTR<sup>®</sup> and MEM series. The compromise with the MEA silicone material is that it has less damping than the previous series. This results in typical loss factors in the range of 0.23 - Resonant Transmissibility of approximately 5.0. The MEA silicone also shows less drift than the standard BTR series elastomer.

**"MEE"** — This is another specialty silicone elastomer which was part of the development of materials for low temperature service. It has excellent consistency over a very broad temperature range—even better than the MEA material described above. The compromise with this elastomer is its low damping level. The typical loss factor for MEE is approximately 0.11 which results in resonant transmissibility in the range of 9.0. The low damping does give this material the desirable feature of providing excellent high frequency isolation characteristics along with its outstanding temperature stability.

With the above background, some of the properties of these elastomers, as they apply to the application of Lord isolators, will be presented. As with metals, elastomers have measureable modulus properties. The stiffness and damping characteristics of isolators are directly proportional to these moduli and vary as the moduli vary.

**Strain, Temperature and Frequency Effects** — The engineering properties of elastomers vary with strain (the amount of deformation due to dynamic disturbance), temperature and the frequency of the dynamic disturbance. Of these three effects, frequency typically is the least and, for most isolator applications, can normally be neglected. Strain and temperature effects must be considered.

**Strain Sensitivity** — The general trend of dynamic modulus with strain is that modulus decreases with increasing strain. This same trend is true of the damping modulus. The ratio of the damping modulus to dynamic elastic modulus is approximately equal to the loss factor for the elastomer. The inverse of this ratio may be equated to the expected resonant transmissibility for the elastomer. This may be expressed as:

$$\frac{G''}{G'} \cong \eta$$
$$\frac{G'}{G''} \cong T_{\mathbf{R}}$$

Where: G' is dynamic modulus (psi)

- G " is damping (loss) modulus (psi)
- $\eta$  is loss factor
- $T_R$  is resonant transmissibility

more exactly:

$$\mathbf{T}_{\mathbf{R}} = \sqrt{\frac{1+\eta^2}{\eta^2}}$$

In general, resonant transmissibility varies only slightly with strain while the dynamic stiffness of an isolator may, depending on the elastomer, vary quite markedly with strain.

Figure 5 presents curves which depict the variation of the dynamic modulus of various elastomers which may be used in vibration isolators as related to the dynamic strain across the elastomer. These curves may be used to approximate the change in dynamic stiffness of an isolator due to the dynamic strain across the elastomer. This is based on the fact that the dynamic stiffness of an isolator is directly proportional to the dynamic modulus of the elastomer used in it. This relationship may be written as:

$$\mathbf{K}' = \frac{\mathbf{A}\mathbf{G}'}{\mathbf{t}}$$

Where: K' is dynamic shear stiffness (lb/in)

G' is dynamic shear modulus of the elastomer (psi)

- t is elastomer thickness (in)
- A is load area of the elastomer  $(in^2)$



FIGURE 5 TYPICAL DYNAMIC ELASTIC MODULUS VALUES FOR LORD VIBRATION ISOLATOR ELASTOMERS

This variation may be used to calculate the change in a dynamic system's natural frequency from the equation:

$$f_n = 3.13 \sqrt{\frac{K'_T}{W}}$$

isolators

Where:  $f_n$  is system natural frequency(Hz)

As there is a change in dynamic modulus, there is a variation in damping due to the effects of strain in elastomeric materials. One indication of the amount of damping in a system is the resonant transmissibility of that system. Figure 6 shows the variation in resonant transmissibility due to changes in vibration input for the elastomers typically used in Lord military electronics isolators.



FIGURE 6 TYPICAL RESONANT TRANSMISSIBILITY VALUES FOR LORD VIBRATION ISOLATOR ELASTOMERS

The data presented in Figures 5 and 6 lead to some conclusions about the application of vibration isolators. The following must be remembered when analyzing or testing an isolated system:

- It is important to specify the dynamic conditions under which the system will be tested.
- The performance of the isolated system will change if the dynamic conditions (such as vibration input) change.
- The change in system performance due to changing dynamic environment may be estimated with some confidence.

**Temperature Sensitivity** — Temperature, like strain, will affect the performance of elastomers and the systems in which elastomeric isolators are used. Figures 7 and 8 show the variations of dynamic modulus and resonant transmissibility with temperature and may be used to estimate system performance changes as may Figures 5 and 6 in the case of strain variation.



FIGURE 7 TYPICAL TEMPERATURE CORRECTIONS FOR LORD VIBRATION ISOLATOR ELASTOMERS



FIGURE 8 TRANSMISSIBILITY VS. TEMPERATURE FOR LORD VIBRATION ISOLATOR ELASTOMERS

# Modifications to Theory Based on the Real World

It should be apparent from the preceding discussion that the basic assumption of linearity in dynamic systems must be modified when dealing with elastomeric vibration isolators. These modifications do affect the results of the analysis of an isolated system and should be taken into account when writing specifications for vibration isolators. It should also be noted that similar effects of variation with vibration level have been detected with "metal mesh" isolators. Thus, care must be exercised in applying them. The amount of variability of these isolators is somewhat different than with elastomeric isolators and depends on too many factors to allow simple statements to be made.

The following discussion will be based on the properties of elastomeric isolators.

## Static Stiffness versus Shock Stiffness versus

**Vibration Stiffnesses** — Because of the strain and frequency sensitivity of elastomers, elastomeric vibration and shock isolators perform quite differently under static, shock or vibration conditions.

The equation:

$$d_{static} = \frac{9.8}{f_n^2}$$

#### Where $d_{static}$ is the "static deflection" of the system (in) and $f_n$ is the system natural frequency (Hz)

DOES NOT HOLD for elastomeric vibration/shock isolators. The static stiffness is typically less than the dynamic stiffness for these materials. To say this another way, the static deflection will be higher than expected if it were calculated, using the above formula, based on a vibration or shock test of the system.

Similarly, neither the static nor the vibration stiffness of such devices is applicable to the condition of shock disturbances of the system. It has been found empirically that:

#### $K_{shock}^{\,\prime}\cong 1.4K_{\,static}$

The difference in stiffness between vibration and static conditions depends on the strain imposed by the vibration on the elastomer. Figure 5 shows where the static modulus will lie in relation to the dynamic modulus for some typical elastomers at various strain levels.

What this means to the packaging engineer or dynamicist is that one, single stiffness value cannot be applied to all conditions and that the dynamic to static stiffness relationship is dependent on the particular isolator being considered. What this means to the isolator designer is that each condition of use must be separately analyzed with the correct isolator stiffness for each condition.

**Shock Consideration** — As stated in the previous discussion, shock analyses for systems using elastomeric isolators should be based on the guideline that the isolator stiffness will be approximately 1.4 times the static stiffness. In addition to this, it must be remembered that there must be enough free deflection in the system to allow the shock energy to be stored in the isolators. If the system should bottom, the "g" level transmitted to the mounted equipment will be much higher than would be calculated. In short, the system must be allowed to oscillate freely once it has been exposed to a shock disturbance to allow their y to be applied appropriately. Figure 9 shows this situation schematically.

In considering the above, several items should be noted:

- Damping in the system will dissipate some of the input energy and the peak transmitted shock will be slightly less than predicted based on a linear, undamped system.
- "τ" is the shock input pulse duration (seconds)
- "t<sub>n</sub>" is one-half of the natural period of the system (seconds)
- There must be enough free deflection allowed in the system to store the energy without bottoming (snubbing). If this is not considered, the transmitted shock may be significantly higher than calculated and damage may occur in the mounted equipment.

**Vibration Considerations** — The performance of typical elastomeric isolators changes with changes in dynamic input—the level of vibration to which the system is being subjected. This is definitely not what most textbooks on vibration would imply. The strain sensitivity of the elastomers causes the dynamic characteristics to change.

Figure 10 is representative of a model of a vibratory system proposed by Professor Snowdon of Penn State University in his book, "Vibration and Shock in Damped Mechanical Systems." This model recognized the changing properties of elastomers and the effects of these changes on the typical vibration response of an isolated system. These effects are depicted in the comparison of a theoretically calculated transmissibility response curve to one resulting from a test of an actual system using elastomeric isolators. RPM Mechanical Inc Website: ww



FIGURE 9

Website: www.rpmmech.com

#### The Real World

The majority of vibration and shock isolators are those utilizing elastomeric elements as the source of compliance and damping to control system responses.



FIGURE 10

 $G^* = G' + jG^{\uparrow}$ or  $G^* = G'(1 + j\eta)$ 

Where ""
"
is loss factor

$$\eta \cong \frac{\mathbf{G}''}{\mathbf{G}'} \cong 2\zeta$$

G<sup>↑</sup> is Damping Modulus (psi) G' is Dynamic Modulus (psi) and ζ is damping factor (dimensionless)

Using this model, we may express the absolute transmissibility of the system as:

ì

$$T_{ABS} = \frac{\sqrt{1 + \eta^2}}{\sqrt{[1 - r^2 \frac{G'}{G_{n'}}] 2 + \eta^2}}$$

# Where $G_n'$ is Dynamic Modulus (psi) at the particular vibration condition being analyzed.

The resulting transmissibility curve from such a treatment, compared to the classical, theoretical transmissibility curve, is shown in Figure 11.



FIGURE 11 EFFECT OF MATERIAL SENSITIVITY ON TRANSMISSIBILITY RESPONSE

Two important conclusions may be reached on the basis of this comparison:

- 1. The "crossover" point of the transmissibility curve  $(T_{ABS} = 1.0)$  occurs at a frequency higher than D times the natural frequency which is what would be expected based on classical vibration theory. This crossover frequency will vary depending on the type of vibration input and the temperature at which the test is being conducted.
- 2. The degree of isolation realized at high frequencies  $(T_{ABS} < 1.0)$  will be less than calculated for an equivalent level of damping in a classical analysis.

This slower "roll-off" rate  $(\frac{db}{octave})$  will depend,

also, on the type of elastomer, level and type of input and temperature.

In general, a constant amplitude sinusoidal vibration input will have less effect on the transmissibility curve than a constant 'g' (acceleration) vibration input. The reason is that, with increasing frequency, the strain across the elastomer is decreasing more rapidly with the constant 'g' input than with a constant amplitude input. Remembering the fact that decreasing strain causes increasing stiffness in elastomeric isolators, this means that the crossover frequency will be higher and the roll-off rate will be lower for a constant 'g' input than for a constant amplitude input. Figure 12 is representative of these two types of vibration inputs as they might appear in a test specification.



No general statement of where the effects of random vibration will lead in relationship to a sinusoidal constant 'g' or constant amplitude vibration input can be made. However, the effects will be similar to a sinusoidal vibration since random vibrations typically produce lower strains across isolators as frequency increases. There may be some exceptions to this statement. The section titled, "Determining Necessary Characteristics of Vibration/Shock Isolator" provides guidance as to how to apply the properties of elastomers to the various conditions which may be specified for a typical installation requiring isolators.

#### Data Required to Select or Design a Vibration/

**Shock Isolator** — As with any engineering activity, the selection or design of an isolator is only as good as the information on which that selection or design is based. Figure 13 is an example of one available Lord checklist for isolator applications — Document number SI-6106.

If the information on this checklist is provided, the selection of an appropriate isolator can be aided greatly, both in timeliness and suitability.

Section I provides the information about the equipment to be mounted (its size, weight and inertias) and the available space for the isolation system to do its job. This latter item includes isolator size and available sway space for equipment movement.

Section II tells the designer what the dynamic disturbances are and how much of those disturbances the equipment can withstand. The difference is the function of the isolation system.

It is important to note here that the random vibration must be provided as a power spectral density versus frequency tabulation or graph, not as an overall "g<sub>rms</sub>" level, in order to allow analysis of this condition. Also, note that the U.S. Navy "high impact" shock test is required by specification MIL-S-901 for shipboard equipment.

Section III contains space for descriptions of any special environmental exposures which the isolators must withstand. Also, for critical applications, such as gyros, optics and radar isolators, the requirements for control of angular motion of the isolated equipment are requested. In such cases, particular effort should be made to keep the elastic center of the isolation system and the center of gravity of the equipment at the same point. The vibration isolators may have their dynamic properties closely matched in order to avoid the introduction of angular errors due to the isolation system itself.

All of the information listed on the checklist shown in Figure 13 is important to the selection of a proper vibration isolator for a given application. As much of the information as possible should be supplied as early as possible in the design or development stage of your equipment. Of course, any drawings or sketches of the equipment and the installation should also be made available to the vibration/shock analyst who is selecting or designing isolators.

#### **Determining Necessary Characteristics** of a Vibration/Shock Isolator

The fragility of the equipment to be isolated is typically the determining factor in the selection or design of an isolator. The critical fragility level may occur under vibration conditions or shock conditions. Given one of these starting points, the designer can then determine the dynamic properties required of isolators for the application. Then, knowing the isolator required, the designer may estimate the remaining dynamic and static performance properties of the isolator and the mounted system.

The following sections will present a method for analyzing the requirements for an isolation problem and for selecting an appropriate isolator.

#### Sinusoidal Vibration Fragility as the Starting Point

— A system specification, equipment operation requirements or a known equipment fragility spectrum may dictate what the system natural frequency must, or may, be. Figure 14 shows a fictitious fragility curve superimposed on a typical vibration input curve. Isolation system requirements may be derived from this information.

**RPM Mechanical Inc** 

Website: www.rpmmech.com

### SAMPLE

## Engineering Data For Vibration and Shock Isolator Questionnaire

| For<br>in a<br>Yo | r actu<br>as mu<br>u ma | ual questionnaire, see page 99. Please fill<br>uch detail as possible before contacting Lord.<br>by mail, fax or e-mail this completed form. | For Technical Assistance, Contact: Application Support,<br>Aerospace Engineering, Lord Corporation,<br>Mechanical Products Division, 2000 W. Grandview Blvd.,<br>Erie, PA 16514: Phone: 814/868-0924, Ext. 6611 or 6497: |  |  |  |  |  |  |
|-------------------|-------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|--|--|--|--|
| I.                | Ph                      | nysical Data                                                                                                                                 | FAX: 814/864-5468; E-mail: apsupport@lord.com                                                                                                                                                                            |  |  |  |  |  |  |
|                   | A.                      | Equipment weight                                                                                                                             |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   | B.                      | 3. C.G. location relative to mounting points                                                                                                 |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   | C.                      | C. Sway space                                                                                                                                |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   | D.                      | 0. Maximum mounting size                                                                                                                     |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   | E.                      | . Equipment and support structure resonance frequencies                                                                                      |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   | F.                      | Moment of inertia through C.G. for major axes (necessary for natural frequency and coupling calculations)                                    |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   |                         | I xx I yy I zz                                                                                                                               |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   | G.                      | G. Fail-safe installation required? Yes 🗌 No 🗌                                                                                               |                                                                                                                                                                                                                          |  |  |  |  |  |  |
| II.               | Dy                      | Dynamics Data                                                                                                                                |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   | A.                      | Vibration requirement:                                                                                                                       |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   |                         | 1. Sinusoidal inputs (specify sweep rate, duration and magnitude or applicable input                                                         |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   |                         | specification curve)                                                                                                                         |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   |                         | 2. Random inputs (specify duration and magnitude $(g^2/Hz)$ applicable input specification curve)                                            |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   | B.                      | 3. Resonant dwell (input & duration)                                                                                                         |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   | C.                      | Shock requirement:                                                                                                                           |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   |                         | 1. Pulse shape pulse p                                                                                                                       | period amplitude                                                                                                                                                                                                         |  |  |  |  |  |  |
|                   |                         | number of shocks per axis maximum output                                                                                                     |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   |                         | 2. Navy hi impact required? (if yes, to what level?)                                                                                         |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   | D.                      | Sustained acceleration: magnitude direction                                                                                                  |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   |                         | Superimposed with vibration? Yes 🗌 No 🗌                                                                                                      |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   | E.                      | Vibration fragility envelope (maximum G vs. frequency preferred) or desired natural frequency and maximum transmissibility                   |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   | F.                      | Maximum dynamic coupling angle                                                                                                               |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   |                         | matched mount required? Yes $\Box$ No $\Box$                                                                                                 |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   | G.                      | Desired returnability                                                                                                                        |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   |                         | Describe test procedure                                                                                                                      |                                                                                                                                                                                                                          |  |  |  |  |  |  |
| Ш                 | l. En                   | vironmental Data                                                                                                                             |                                                                                                                                                                                                                          |  |  |  |  |  |  |
|                   | A.                      | Temperature: Operating                                                                                                                       | Non-operating                                                                                                                                                                                                            |  |  |  |  |  |  |
|                   | В.                      | Salt spray per MIL                                                                                                                           | Humidity per MIL                                                                                                                                                                                                         |  |  |  |  |  |  |
|                   |                         | Sand and dust per MIL                                                                                                                        | Fungus resistance per MIL                                                                                                                                                                                                |  |  |  |  |  |  |
|                   |                         | Oil and/or gas                                                                                                                               | Fuels                                                                                                                                                                                                                    |  |  |  |  |  |  |
|                   | C.                      | Special finishes on components                                                                                                               |                                                                                                                                                                                                                          |  |  |  |  |  |  |



FIGURE 14 EQUIPMENT FRAGILITY VS. VIBRATION INPUT

First, the allowable transmissibility at any frequency may be calculated as the ratio of the allowable output to the specified input.

$$T_{ABS} = \frac{X_o}{X_i} \text{ or } \frac{g_o}{g_i}$$

The frequency at which this ratio is a maximum is one frequency at which the system natural frequency may be placed (assuming that it is greater than approximately 2.5, at some frequency). Another method of placing the system natural frequency is to select that frequency which will allow the isolation of the input over the required frequency range. A good rule of thumb is to select a frequency which is at least a factor of 2.0 below that frequency where the allowable response (output) crosses over — goes below — the specified input curve.

Having determined an acceptable system natural frequency, the system stiffness (spring rate) may be calculated from the following relationship:

$$K'_v = \frac{(f_n)^2(W)}{9.8}$$

Where: K'<sub>V</sub> is the total system dynamic stiffness (lb/in) at the specified vibration input

- f<sub>n</sub> is the selected system natural frequency (Hz)
- W is the isolated equipment weight (lbs)

An individual isolator spring rate may then be determined by dividing this system spring rate by the allowable, or desired number of isolators to be used. The appropriate isolator may then be selected based on the following factors:

- specified vibration input at the desired natural frequency of the system
- · static load supported per isolator
- allowable system transmissibility
- environmental conditions (temperature, fluid exposure, etc.)

Once a particular isolator has been selected, the properties of the elastomer in the isolator may be used to estimate the performance of the isolator at other conditions of use, such as other vibration levels, shock inputs, steady state acceleration loading and temperature extremes. The necessary elastomer property data are found in Figures 5, 6, 7 and 8.

If the vibration input in the region of the required natural frequency is specified as a constant acceleration—constant 'g'—it may be converted to a motion input through the equation:

$$X_i = \frac{g_i}{(0.051)(f_n)^2}$$

Where: X<sub>i</sub> is vibratory motion (inches, double amplitude)

- g<sub>i</sub> is specified vibratory acceleration input (g)
- $\mathbf{f}_{\mathbf{n}}$  is the desired system natural frequency (Hz)

Of course, this equation may be used to convert constant acceleration levels to motions at any frequency. It is necessary to know this vibratory motion input in order to select or design an isolator. Note, that most catalog vibration isolators are rated for some maximum vibration input level expressed in inches double amplitude. Also, the listed dynamic stiffnesses for many standard isolators are given for specific vibration inputs. This information provides a starting point on Figure 5 to allow calculation of the system performance at vibration levels other than that listed for the isolator.

#### **Random Vibration Performance as the Starting**

**Point** — Random vibration is replacing sinusoidal vibration in specifications for much of today's equipment. A good example is MIL-STD-810. Many of the vibration levels in the most recent version of this specification are given in the now familiar format of "power spectral density" plots. Such specifications are the latest attempt to simulate the actual conditions facing sensitive equipment in various installations.

A combination of theory and experience is used in the analysis of random vibration. As noted previously, the random input must be specified in the units of " $g^2/Hz$ "

• required dynamic spring rate

**RPM Mechanical Inc** 

in order to be analyzed and to allow proper isolator selection. The system natural frequency may be determined by a fragility versus input plot of random vibration just as was done and demonstrated in Figure 14 for sinusoidal vibration. Once the required natural frequency is known, the necessary isolator spring rate may again be calculated from the equation:

$$K'_{v} = \frac{(f_{n})^{2}(W)}{9.8}$$

The next steps in determining which isolator may be used are to calculate the allowable transmissibility and the motion at which the isolated system responds at the same natural frequency as when it is subjected to the specified random vibration. The allowable transmissibility, if not already specified, may be calculated from the input vibration and the allowable vibration by using the equation:

$$T_{R} = \sqrt{\frac{S_{o}}{S_{i}}}$$

Where, T<sub>R</sub> is the resonant transmissibility

- (dimensionless)  $S_0$  is output random vibration (g<sup>2</sup>/Hz)  $S_i$  is input random vibration (g<sup>2</sup>/Hz)

A sinusoidal vibration input, acceleration or motion, at which the system will respond at approximately the same natural frequency with the specified random vibration may be calculated in the following manner.

Step 1. The analysis of random vibration is made on the basis of probability theory. The one sigma  $(1\sigma)$ RMS acceleration response may be calculated from the equation:

$$g_{0RMS} = \sqrt{(\pi/2)(S_i)(f_n)T_R}$$

#### Where, $g_0$ RMS is the 1 $\sigma$ RMS acceleration response (g)

- $S_i$  is input random vibration (g2/Hz)
- $\mathbf{T}_{\mathbf{R}}$  is allowable resonant transmissibility

is desired natural frequency (Hz)

**Step 2.** It has been found empirically that elastomeric isolators typically respond at a  $3\sigma$  vibration level. Thus, the acceleration vibration level at which the system will respond at approximately the same natural frequency as with the specified random level may be found to be:

$$g_{3\sigma} = 3\sqrt{(\pi/2)(S_i)(f_n)T_R}$$

**Step 3.** The above is response acceleration. To find the input for this condition of response, we simply divide by the resonant transmissibility.

$$g_i = \frac{g_{3\sigma}}{T_R}$$

**Step 4.** Finally, we apply the equation from a previous section to calculate the motion input vibration equivailent to this acceleration at the system natural frequency:

$$X_{i} = \frac{g_{i}}{(0.051)(f_{n})^{2}}$$

#### Note that X<sub>i</sub> is in units of inches double amplitude.

Step 5. The analysis can now follow the scheme of previous calculations to find the appropriate isolator and then analyze the shock, static and temperature performance of the isolator.

Shock Fragility as the Starting Point —If the fragility of the equipment in a shock environment is the critical requirement of the application, the natural frequency of the system will depend on the required isolation of the shock input.

**Step 1.** Calculate the necessary shock transmissibility

$$T_S = \frac{g_0}{g_i}$$

Where T<sub>s</sub> is shock transmissibility (dimensionless) g<sub>0</sub> is equipment fragility (g)

is input shock level (g)

**Step 2.** Calculate the required shock natural frequency. This depends on the shape of the shock pulse.

The following approximate equations may be used only for values of  $T_s < 1.0$ :

| Pulse Shape   | Transmissibility Equation |
|---------------|---------------------------|
| Half Sine     | $Ts \cong 4(f_n)(t_o)$    |
| Square Wave   | $Ts \cong 6(f_n)(t_o)$    |
| Triangular    | $Ts \cong 3.1(f_n)(t_o)$  |
| Ramp or Blast | $Ts \cong 3.2(f_n)(t_o)$  |

# $\begin{array}{lll} \mbox{Where} & T_s & \mbox{is shock transmissibility} \\ f_n & \mbox{is shock natural frequency} \\ t_o & \mbox{is shock pulse length (seconds)} \end{array}$

Remember, that the system natural frequency under a shock condition will typically be different from that under a vibration condition for systems using elastomeric vibration isolators.

**Step 3.** Calculate the required deflection to allow this level of shock protection by the equation:

$$\mathbf{d_s} = \frac{\mathbf{g_0}}{(0.102)(\mathbf{f_n}^2)}$$

#### Where $d_s$ is shock deflection (inches Single Amplitude) $g_o$ is shock response or equipment fragility (g) $f_n$ is shock natural frequency (Hz)

**Step 4.** Calculate the required dynamic spring rate necessary under the specified shock condition from the equation:

$$\mathbf{K}_{\mathbf{S}}' = \frac{(\mathbf{f}_{\mathbf{n}})^2 \mathbf{W}}{9.8}$$

# WhereK'sis dynamic stiffness (lb/in)fnis shock natural frequency (Hz)Wis supported weight (lbs)

**Step 5.** Select the proper isolator from those available in the product section, that is, one which has the required dynamic stiffness  $(\mathbf{K'}_v)$ , will support the specified load and will allow the calculated deflection  $(\mathbf{d_s})$  without bottoming during the shock event.

**Step 6.** Determine the dynamic stiffness  $(\mathbf{K'}_v)$  of the chosen isolator, at the vibration levels specified for the application, by applying Figure 5 with the knowledge that dynamic spring rate is directly proportional to dynamic modulus (**G'**) and by working from a known dynamic stiffness of the isolator at a known dynamic motion input.

**Step 7.** Calculate system natural frequencies under specified vibration inputs from the equation:

$$f_n = 3.13 \sqrt{\frac{K'_v}{W}}$$

$$\begin{array}{ll} \text{Where } f_n & \text{is vibration natural frequency (Hz)} \\ \text{K'}_v & \text{is isolator dynamic stiffness at the} \\ & \text{specified vibration level (lbs/in)} \\ \text{W} & \text{is the supported weight (lbs)} \end{array}$$

Note that the stiffness and supported weight must be considered on the same terms, i.e., if the stiffness is for a single mount, then the supported weight must be that supported on one mount. Once the system natural frequency is calculated, the system should be analyzed to determine what effect this resonance will have on the operation and/or protection of the equipment.

**Step 8.** Estimate the static stiffness of the isolators from the relationship:

$$K\cong \frac{K_s'}{1.4}$$

#### Where K is static stiffness (lbs/in) K'<sub>s</sub> is shock dynamic stiffness (lbs/in)

Then, check the deflection of the system under the 1g load and under any steady-state (maneuver) loads from the equation:

$$\mathbf{d_s} = \frac{\mathbf{gW}}{\mathbf{K}}$$

Where d<sub>s</sub> is static deflection (inches) g is the number of g's loading being imposed Wis the supported load (lbs) K is static spring rate (lbs/in)

Be sure that the chosen isolator has enough deflection capability to accommodate the calculated motions without bottoming. If the vibration isolation function and steady state accelerations must be imposed on the system simultaneously, the total deflection capability of the isolator must be adequate to allow the deflections from these two sources combined. Thus,

$$d_{total} = d_v + d_s$$
  
where  $d_v = \frac{x_i}{2} T_R$ 

- and where x<sub>i</sub> is input vibration motion at resonance (inches double amplitude)
  - d<sub>v</sub> is deflection due to vibration (inches single amplitude)
  - T<sub>R</sub> is resonant transmissibility
  - d<sub>s</sub> is static deflection per the above equation (inches)

**Types of Isolators and Their Properties** — There are a number of different types of isolators, based on configuration, which may be applied in supporting and protecting various kinds of equipment. Depending on the severity of the application and on the level of protection required for the equipment, one or another of these mounting types may be applied.

Figures 15, 16 and 17 show some of the most common "generic" configurations of vibration isolators and the characteristic load versus deflection curves for the simple shear mounting and the "buckling column" types of isolators. In general, the fully bonded or holder types of isolators are used for more critical equipment installations because these have superior performance characteristics as compared to the center bonded or unbonded configurations. The buckling column type of isolator is useful in applications where high levels of shock must be reduced in order to protect the mounted equipment. Many aerospace equipment isolators are of the conical type because they are isoelastic.

In order of preference for repeatability of performance the rank of the various isolator types is:

- 1. Fully Bonded
- 2. Holder Type
- 3. Center Bonded
- 4. Unbonded

In reviewing the standard lines of Lord isolators, the STANDARD AVIONICS (AM), PEDESTAL (PS), PLATEFORM (100,106,150,156), HIGH DEFLEC-TION (HDM) and MINIATURE (MAA) mounts are in the fully bonded category. The BTR (HT) mounts are the only series in the holder type category. The MINIATURE (MCB) series of isolators is the offering in the center bonded type of mount. The MINIATURE GROMMETS (MGN and MGS) are in the unbonded mount category. In total, these standard offerings from Lord cover a wide range of stiffnesses and load ratings to satisfy the requirements of many vibration and shock isolation applications. In some instances, there may be a need to match the dynamic stiffness and damping characteristics of the isolators which are to be used on any particular piece of equipment. Some typical applications of matched sets of isolators are gyros, radars and optics equipment. For these applications, the fully bonded type of isolator construction is highly recommended. The dynamic performance of these mounts is much more consistent than other types. Dynamically matched isolators are supplied in sets but are not standard since matching requirements are rarely the same for any two applications.









**Sample Application Analysis** — Figure 18 is a completed checklist of information for a fictitious piece of Avionics gear installed in an aircraft environment. The following section will demonstrate how the foregoing

theory and data may be applied to the selection of a standard Lord mount.

# CONSIDER SINUSOIDAL VIBRATION REQUIREMENTS

From the checklist, it is noted that the desired system natural frequency is 32 Hz with a maximum allowable transmissibility of 4.0, or less.

Step 1. Determine the required dynamic spring rate:

$$K'_{v} = \frac{(f_{n})^{2}(W)}{9.8}$$
$$f_{n} = 32 \text{ Hz}$$
$$W = 12 \text{ lbs}$$

$$K'_v = \frac{(32)^2(12)}{9.8} = 1254$$
 lbs/in

Note that this figure is the total system spring rate since the weight used in the calculation was the total weight of the supported equipment. The checklist indicates that four (4) isolators will be used to support this unit. Thus, the required isolator is to have a dynamic stiffness of:

$$K'_{v} = \frac{1254}{4} = 314$$
 lbs/in/isolator

at the vibration input of 0.036 inch double amplitude as specified in section II.A.1 of the checklist.

Step 2. Make a tentative isolator selection.

Thus far, it is known that:

- 1. The isolator must have a dynamic spring rate of 314 lbs/in.
- 2. The supported static load per isolator is 3 pounds.
- 3. The material, or construction, of the isolator must provide enough damping to control resonant transmissibility to 4.0 or less.
- 4. There is no special environmental resistance required.

Choosing a relatively small isolator available from those which meet the above requirements, the AM003-7, in BTR<sup>®</sup> elastomer, is selected from the product data section. The analysis now proceeds to consideration of other specified conditions.

## SAMPLE

### Engineering Data For Vibration and Shock Isolators Questionnaire

| Fo<br>in<br>Yo | or act<br>as m<br>ou ma | tual questionnaire, see page 99. Please f<br>nuch detail as possible before contacting<br>ay mail, fax or e-mail this completed form | ill<br>Lord.<br>1.                                                                           | For Technical Assis<br>Support, Aerospace<br>Mechanical Produc | tance, Contact:<br>Engineering, I<br>ts Division, 20 | Application<br>Lord Corporation,<br>00 W. Grandview |  |  |  |
|----------------|-------------------------|--------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------|----------------------------------------------------------------|------------------------------------------------------|-----------------------------------------------------|--|--|--|
| I.             | Ph                      | ysical Data                                                                                                                          | Blvd., Erie, PA 16514; Phone: 814/868-0924,<br>Ext. 6611 or 6497; FAX: 814/864-5468; E-mail: |                                                                |                                                      |                                                     |  |  |  |
|                | A.                      | Equipment weight <u>12 lbs.</u>                                                                                                      |                                                                                              | apsupport@lord.co                                              | m                                                    |                                                     |  |  |  |
|                | В.                      | C.G. location relative to mounting point                                                                                             | ts                                                                                           | Geometric Ce                                                   | nter                                                 |                                                     |  |  |  |
|                |                         | Four Mounts Desir                                                                                                                    | red                                                                                          |                                                                |                                                      |                                                     |  |  |  |
|                | C.                      | Sway space ± 0.32"                                                                                                                   |                                                                                              |                                                                |                                                      |                                                     |  |  |  |
|                | D.                      | D. Maximum mounting size 1" High x 2" Long x 2" Wide                                                                                 |                                                                                              |                                                                |                                                      |                                                     |  |  |  |
|                | E.                      | E. Equipment and support structure resonance frequencies 400 Hz                                                                      |                                                                                              |                                                                |                                                      |                                                     |  |  |  |
|                | F.                      | Moment of inertia through C.G. for maj calculations)                                                                                 | requency and c                                                                               | oupling                                                        |                                                      |                                                     |  |  |  |
|                |                         | (unknown) I xx                                                                                                                       | I yy                                                                                         |                                                                | _ I zz                                               | <u> </u>                                            |  |  |  |
|                | G.                      | Fail-safe installation required? Yes                                                                                                 | 🗌 No                                                                                         | X                                                              |                                                      |                                                     |  |  |  |
| П.             | Dy                      | Dynamics Data                                                                                                                        |                                                                                              |                                                                |                                                      |                                                     |  |  |  |
|                | A.                      | Vibration requirement:                                                                                                               |                                                                                              |                                                                |                                                      |                                                     |  |  |  |
|                |                         | 1. Sinusoidal inputs (specify sweep rate, duration and magnitude or applicable input                                                 |                                                                                              |                                                                |                                                      |                                                     |  |  |  |
|                |                         | specification curve)                                                                                                                 |                                                                                              |                                                                |                                                      |                                                     |  |  |  |
|                |                         | 036" D.A. 5                                                                                                                          | <u>5 to 52 Hz</u>                                                                            | ; 5G, 52 to 500 Hz                                             |                                                      |                                                     |  |  |  |
|                |                         | input specificat                                                                                                                     | tion curve)                                                                                  |                                                                |                                                      |                                                     |  |  |  |
|                | B.                      | B. Resonant dwell (input & duration)036" D.A. 1/2 hr. per Axis                                                                       |                                                                                              |                                                                |                                                      |                                                     |  |  |  |
|                | C.                      | Shock requirement:                                                                                                                   |                                                                                              |                                                                |                                                      |                                                     |  |  |  |
|                |                         | 1. Pulse shape <u>Half Sine</u> pu                                                                                                   | Ise period                                                                                   | 1 <u>11ms</u>                                                  | _ amplitude                                          | 15G                                                 |  |  |  |
|                |                         | number of shocks per axis3/A:                                                                                                        | xis                                                                                          | maximum o                                                      | output                                               | N/A                                                 |  |  |  |
|                |                         | 2. Navy hi impact required? N/A                                                                                                      | (if ye                                                                                       | es, to what level?)                                            |                                                      |                                                     |  |  |  |
|                | D.                      | Sustained acceleration: magnitude                                                                                                    |                                                                                              | 3G                                                             | _ direction                                          | all directions                                      |  |  |  |
|                |                         | Superimposed with vibration? Yes                                                                                                     | x No                                                                                         |                                                                |                                                      |                                                     |  |  |  |
|                | E.                      | Vibration fragility envelope (maximum                                                                                                | G vs. free                                                                                   | quency preferred) or d                                         | esired natural f                                     | requency and                                        |  |  |  |
|                |                         | maximum transmissibility 32 Hz with T less than 4                                                                                    |                                                                                              |                                                                |                                                      |                                                     |  |  |  |
|                | F.                      | Maximum dynamic coupling angle                                                                                                       |                                                                                              | N.A.                                                           |                                                      |                                                     |  |  |  |
|                |                         | matched mount required? Yes                                                                                                          | 🗌 No                                                                                         |                                                                |                                                      |                                                     |  |  |  |
|                | G.                      | Desired returnability                                                                                                                |                                                                                              | N.A.                                                           |                                                      |                                                     |  |  |  |
|                |                         | Describe test procedure                                                                                                              |                                                                                              | N.A.                                                           |                                                      |                                                     |  |  |  |
| Ш              | l. En                   | vironmental Data                                                                                                                     |                                                                                              |                                                                |                                                      |                                                     |  |  |  |
|                | А.                      | Temperature: Operating <u>+30</u>                                                                                                    | $0^{\circ}$ to +120                                                                          | <sup>o</sup> F Non-opera                                       | ting40° t                                            | o +160°F                                            |  |  |  |
|                | B.                      | Salt spray per MIL 810                                                                                                               | C                                                                                            | _ Humidity per MIL _                                           |                                                      | 810C                                                |  |  |  |
|                |                         | Sand and dust per MIL 810                                                                                                            | )C                                                                                           | _ Fungus resistance p                                          | er MIL                                               | 810C                                                |  |  |  |
|                |                         | Oil and/or gas N.A                                                                                                                   | ٩.                                                                                           | Fuels                                                          |                                                      | N.A.                                                |  |  |  |
|                | C.                      | Special finishes on components                                                                                                       |                                                                                              | N.A.                                                           |                                                      |                                                     |  |  |  |
|                |                         | 1 f                                                                                                                                  |                                                                                              |                                                                |                                                      |                                                     |  |  |  |

#### **Consider Random Vibration Requirements**

Step 1. Calculate a sinusoidal motion input at the desired natural frequency with the specified random vibration input and compare it to the specified sine vibration. Both the maximum motion and the input motion which would cause the isolator to respond at approximately the same natural frequency as the random vibration should be calculated. The maximum is calculated to check that the selected isolator will have enough deflection capability and the resonant motion is calculated to verify the stiffness of the required isolator at the actual input at which it will respond to the random vibration.

Per the previously presented material, the isolator should respond at a  $3\sigma$  equivalent acceleration calculated on the basis of the specified random vibration at the desired natural frequency. This level will determine, in part, the isolator choice. The calculation is made as follows:

$$g_{03\sigma} = 3\sqrt{(\pi/2)(S_i)(f_n)(T_R)}$$

In which:  $S_i = 0.04 \text{ g2/Hz}$   $T_R = 2.9 \text{ (per Figure 6 for BTR<sup>®</sup> at typical operating strain)}$   $f_n = 32 \text{ Hz}$   $g_{03\sigma} = 3\sqrt{(\pi/2)(0.04)(32)(2.9)}$  $g_{03\sigma} = 7.24 \text{ g}$ 

This is the acceleration <u>response</u> at the desired natural frequency of 32 Hz. The motion across the isolator due to this response may be calculated as:

$$x_{03\sigma} = g_{03\sigma} / (0.051)(f_n^2)$$
  

$$x_{03\sigma} = 7.24 / (0.051)(32^2)$$
  

$$x_{03\sigma} = 0.139 \text{ inch double amplitude}$$

The ultimately selected isolator must have enough deflection capability to allow this motion without bottoming (snubbing). The input acceleration is calculated as:

$$g_{i3\sigma} = g_{03\sigma} / T_R$$
  
 $g_{i3\sigma} = 7.24 / 2.9$   
 $g_{i3\sigma} = 2.5g$ 

and the input motion as:

$$\begin{aligned} x_{i_{3\sigma}} &= g_{i_{3\sigma}} / (0.051) (f_n^2) \\ x_{i_{3\sigma}} &= 2.5 / (0.051) (32^2) \\ x_{i_{3\sigma}} &= 0.048 \text{ inch double amplitude} \end{aligned}$$

This vibration level is higher than the capability of the tentatively selected AM003-7. To remain with a relatively small isolator which will support 3 pounds, withstand the 0.047 inch double amplitude sine vibration and provide an approximate stiffness of 314 lb/in per mounting point, a selection from either the AM002 or AM004 series appears to be best.

Since none of the single isolators provides enough stiffness, a back to back (parallel) installation of a pair of isolators at each mounting point is suggested. Since the AM002 is smaller than the AM004, and is rated for 0.06 inch double amplitude maximum input vibration, the selection of the AM002-8 isolator is made. A pair of the AM002-8 isolators will provide a stiffness of 346 lb/inch (two times 173 per the stiffness chart in the product section). This stiffness would provide a slightly higher natural frequency than desired. However, there is a correction to be made, based on the calculated vibration input.

The stiffnesses in the AM002 product chart are based on an input vibration of 0.036 inch double amplitude. Figure 5 shows that the modulus of the BTR<sup>®</sup> elastomer is sensitive to the vibration input. The modulus is directly proportional to the stiffness of the vibration isolator. Thus, the information of Figure 5 may be used to estimate the performance of an isolator at an "off spec" condition. A simple graphical method may be used to estimate the performance of an isolator at such a condition.

Knowing the geometry of the isolator, the strain at various conditions may be estimated. The modulus versus strain information of Figure 5 and the knowledge of the relationship of modulus to natural frequency (via the stiffness of the isolator) are used to construct the graph of the isolator characteristic. The equation for calculation of the  $3\sigma$  random equivalent input at various frequencies has been shown previously. The crossing point of the two lines on the graph shown in Figure 19 is a reasonable estimate for the response natural frequency of the selected isolator under the specified 0.04 g2/Hz random vibration.

The intersection of the plotted lines in Figure 19 is at a frequency of approximately 32 to 33 Hz, and at an input vibration level of approximately 0.047 inch DA. This matches the desired system natural frequency and confirms the selection of the AM002-8 for this application. In all, eight (8) pieces of the AM002-8 will be used to provide the 32 Hz system natural frequency, while supporting a total 12 lb unit, under the specified random vibration of 0.04 g2/Hz. The eight isolators will be installed in pairs at four locations. With this portion of the analysis complete, the next operating condition - shock - is now considered.



#### **Consider Shock Requirements**

The specified shock input is a 15g, 11 millisecond, half-sine pulse. From the previously presented theory, an approximation of the shock response may be found through the use of the equation:

$$T_s \cong 4f_n t_o$$

Note that the natural frequency to be used here is the shock natural frequency which may be estimated from the information given in Figure 5. The dynamic modulus for the elastomer used here is approximately 120 psi at a vibration level of 0.036 inch double amplitude and the static modulus is approximately 80 psi. From this information, the static stiffness of the isolator may be estimated as follows:

$$K = (\frac{80}{120})(K')$$
$$K = (\frac{80}{120})(\frac{f_n^2 W}{9.8})$$

K = 
$$(\frac{80}{120})(\frac{(32)^2(12)}{9.8})$$
 = 836 lbs/in for the total system

As noted in previous discussion, the shock stiffness is approximately 1.4 times the static stiffness. Thus,

K'shock  $\approx$  (1.4)(836) = 1170 lbs/in total

This makes the shock natural frequency:

$$f_{\text{shock}} = 3.13 \sqrt{\frac{1170}{12}} = 31 \text{ Hz}$$

Thus, the calculation for the shock transmissibility becomes:

$$T_s \cong (4)(31)(.011) = 1.4$$



SINGLE DEGREE OF FREEDOM SYSTEM RESPONSE TO VARIOUS SHOCK PULSES

Since this value is above 1.0, and the equation is only valid up to a value of 1.0, the information of Figure 20 must be used. Use of this graph indicates that the shock transmissibility will be approximately 1.22. Thus, the shock response will be:

$$g_0 = T_s(g_i)$$
  
 $G_0 = (1.22)(15) = 18.3 g$ 

From this response, the next step is to calculate the expected deflection when the selected isolator is subjected to the specified shock input. The equation of interest is:

$$d_{s} = \frac{g_{0}}{(0.102)(f_{n})^{2}}$$

ds = 
$$\frac{18.3}{(0.102)(31)^2}$$
 = 0.19 inch single amplitude

The tentatively selected isolator, AM002-8, is capable of this much deflection without bottoming. Thus, the analysis proceeds to another operating condition.

**Consider "Static" Loading Conditions:** The static loading conditions in an isolator analysis are important from the standpoints of stress and deflection to which the isolator will be exposed. Such conditions are caused by the 1g load which the isolator must support as well as by any maneuver and/or steadystate accelerations, which may be imposed. In the present example, the static system stiffness was calculated as being 836 lbs/in. The deflection of the system at any steady-state "g" loading may be calculated by using the equation:

$$\mathbf{d}_{\text{static}} = \frac{(\mathbf{g})(\mathbf{W})}{\mathbf{K}_{\text{static}}}$$

In the example, the sustained acceleration was specified as being 3g. Thus, the system deflection will be approximately:

$$d_{\text{static}} = \frac{(3)(12)}{836} = 0.043$$
 inch

The selected isolator, AM002-8, is able to accommodate this deflection, even superimposed on the vibration conditions. Finally, none of the environmental conditions shown on the checklist will be of any concern. Thus, this appears to be an appropriate isolator selection. Of course, typical testing of this equipment, supported by the selected isolators, should be conducted to prove the suitability of this system.

The isolators presented in the product portion of this catalog will prove appropriate for many equipment installations. Should one of these products not be suitable, a custom design may be produced. Lord is particularly well equipped to provide engineering support for such opportunities. For contact information, see page 103. The following brief explanation will provide a rough sizing method for an isolator.

Estimating Isolator Size: There will be occasions when custom designs will be required for vibration and shock isolators. It should be remembered that schedule and economy are in favor of the use of the standard isolators shown in the product section here. These products should be used wherever possible. Where these will not suffice, Lord will assist by providing the design of a special mount. The guidelines presented here are to allow the packaging or equipment engineer to estimate the size of the isolator so that the equipment installation can be made with the thought in mind to allow space for the isolators and for the necessary deflection of the system as

supported on them. The final isolator size may be slightly larger or smaller depending on the specifications being imposed.

Figure 21 shows a schematic of a conical isolator, such as may be used for protection of avionic equipment. The two most important parameters in estimating the size of such an isolator are the length of the elastomer wall,  $\mathbf{t}_{\mathbf{R}}$ , and the available load area. For purposes of simplification, a conical angle of 45° is used here. The ratio of axial to radial stiffness depends on this angle.



FIGURE 21 ESTIMATING AVIONICS ISOLATOR SIZE

The elastomer wall length may be estimated based on the dynamic motion necessary for the requirements of the application. This length may be estimated through the following equation:

$$t_{\rm R} = \frac{(x_{\rm i})(T_{\rm R})}{0.30}$$

#### Where $t_R$ is the elastomer wall length (inches) x<sub>i</sub> is the resonant vibration input (inches, double amplitude) T<sub>R</sub> is resonant transmissibility

From the required natural frequency, the necessary dynamic spring rate is known from:

$$K' = \frac{(fn)^2(W)}{9.8} lb/in$$

Where K' is dynamic stiffness (lb/in)

f<sub>n</sub> is desired natural frequency (Hz)
 W is supported weight per isolator (lbs)

For a conical type isolator, the dynamic spring rate/ geometry relationship is:

$$\mathbf{K}' = \frac{(\mathbf{A})(\mathbf{G}')}{\mathbf{t}_{\mathbf{R}}}$$

# Where $t_R$ is the elastomer wall per the above and the area term (A) is estimated as:

$$A\cong 1.4\,\pi(r_2^2-r_1^2)$$

This area term should be determined such that the dynamic stress at resonance is kept below approximately 40 psi.

$$\sigma = \frac{P}{A} \le 40 \text{ psi}$$
  
and  
$$P_{max} \cong (g_i)(T_R)W$$

#### Where g<sub>i</sub> is input 'g' level at resonance T<sub>R</sub> is resonant transmissibility W is supported load per isolator (lbs)

The combination of the elastomer wall length ( $t_R$ ) and load area (A), estimated from the above, and the required attachment features will provide a good estimate of the size of the isolator required to perform the necessary isolation functions. The proper dynamic modulus is then selected for the isolator from an available range of approximately 90 to 250 psi at a 0.036 inch D.A., vibration input.

**Resonant Dwells:** The requirement of a "resonant dwell" of isolated equipment is becoming less common in today's world. However, some projects still have such a requirement and it may be noted that many of the products described in the product sections have been exposed to resonant dwell conditions and have performed very well. Isolators designed to the elastomer wall and load area guidelines given above will survive resonant dwell tests without significant damage for systems with natural frequencies below approximately 65 Hz. Systems higher in natural frequency than this require special consideration and Lord engineers should be consulted. **Environmental Resistance:** Many of the isolators shown in this catalog are inherently resistant to most of the environments (temperature, sand, dust, fungus, ozone, etc.) required by many specifications. The silicone elastomers are all in this category. One particularly critical area is fluid resistance where special oils, fuels or hydraulic fluids could possibly come into contact with the elastomer. Lord engineering should be contacted for an appropriate elastomer selection.

**Testing of Vibration/Shock Isolators:** Lord has excellent facilities for the testing of isolators. Electrodynamic shakers having up to eight thousand pound dynamic force capability are used to test many of the isolators designed or selected for customer use. These shakers are capable of sinusoidal and random vibration testing as well as sine-on-random and random-onrandom conditions. These machines are also capable of many combinations of shock conditions and are supplemented with free-fall drop test machines. Numerous isolator qualification tests have been performed within the test facilities at Lord.

#### **Further Theory**

The preceding discussion presented general theory which is applicable to a broad class of vibration and shock problems. A special class of shock analysis is that which involves drop tests, or specifications, such as with protective shipping containers. This topic is treated in the following pages.